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Fluid Mechanics of the Descemet Membrane Detachment with Spontaneous Reattachment

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ABSTRACT

Descemet membrane detachment (DMD) often happen during cataract surgery. DMD is developed when the membrane separate from the stroma. The separation occur is due to the flow of aqueous humour (AH) from the aqueous chamber into the descrement membrance (DM) space through a break. A mathematical model of buoyancy driven AH flow is developed to explain the mechanisms of the fluid. The existing of temperature different between the cornea and the pupil will drive the fluid to flow due to buoyancy force. The effect of the fluid flow to the deformation of the detach membrane is analyzed. The phenomena of spontaneous reattachment of the DMD will be studied. Numerically, the relevant variables will be computed using finite element method with the aiding of COMSOL Multiphysics. The results have shown that the flowing of AH affect the displacement of the detach membrane and the spontaneous reattachment may indeed occur under certain conditions.

Keywords: Descemet membrane detachment, finite element, Buoyancy force

1. Introduction

The clear dome like surface that exists at the front of a human eye is named as cornea. The cornea covers the iris and the pupil, as shown in Fig. 1. The region that bounded by the cornea, the iris and the pupil is known as the anterior chamber (AC). Three main layers and two auxiliary layers are contained in the cornea, those are the epithelium, the stroma, the endothelium, the Browman layer and the Descemet membrane. Descemet membrane (DM) is the layer lies between the stroma and the endothelium layer of the cornea. Descement membrane detachment (DMD) happens when the DM is separated away from the stroma by the aqueous humour that flows into the space between the membrane and the stroma through a tear or break on the DM. The detachment may have serious adverse effects to the vision function of the human eye. The detachment may have serious adverse effects to the vision function of the human eye. Planar or non-planar, scrolled or non-scrolled and peripheral or peripheral with central cornea involvement are the types of the DMD (see, for example, Menezo et al. (2002) - Potter and Zalatimo (2005)). The DMD cases due to cataract surgery, iridectomy, trabeculectomy, corneal transplantation, deep lamellar keratoplasty, holmium laser sclerostomy, alkali burn and viscocanalostomy have been reported by Mulhern and associates, Potter and Zalatimo, andMulhern et al. (1996) - Unlu and Aksunger (2000). Sevillano and associates Sevillano et al. (2008), had reported the technique of curing of DMD caused by cataract surgery with sulphur hexafluoride injection. Potter and Zalatimo (Potter and Zalatimo (2005)), has presented the case of treating the scrolled DMD by injecting fourteen percent of intracameral perfluoropropane (C3F8) into the AC. Recently, Couch and Baratz (Couch and Barazt (2009)), investigated two cases of delayed bilateral DM and in one eye it was fixed surgically and the other eye improved spontaneously. The authors Couch and Barazt (2009), estimated that the spontaneous reattachment happen may because of the buoyancy effects which cause the aqueous humour (AH) flow in the AC. The spontaneous reattachment of the DM has been supported by some observational and anecdotal evidence (see, for example, Couch and Barazt (2009)-Ismail et al. (2013)). Fitt and Gonzalez (Fitt and Gonzalez (2006)), has showed that under normal conditions the buoyancy effects due to temperature gradient in the AC will enhance the AH to flow. However, the exact mechanisms of the spontaneous reattachment appears not to be known. Therefore, in this study, we intend to apply the fluid mechanical theory to model the flow in AC (developed by Canning et al. (2002)) with the presence of detached

DM in the flow to study the phenomena mathematically in order to explain how and why spontaneous reattachment occurs as noted by Couch and Baratz (Couch and Barazt (2009)). Finite element methods will be applied to analyse the relevant fluid flow equations. The numerical solutions will be computed with the aid of COMSOL Multiphysicals software. Finite element methods is chosen because it is easily applied to objects with complex geometry like the shape of the cornea and having mixed boundary condition and the objects may composed of several different materials in its medium (see, for example, Reddy and Gartling (2010)-Lewis et al. (2008)).

2. Formulation of Problem

2.1 Model Construction

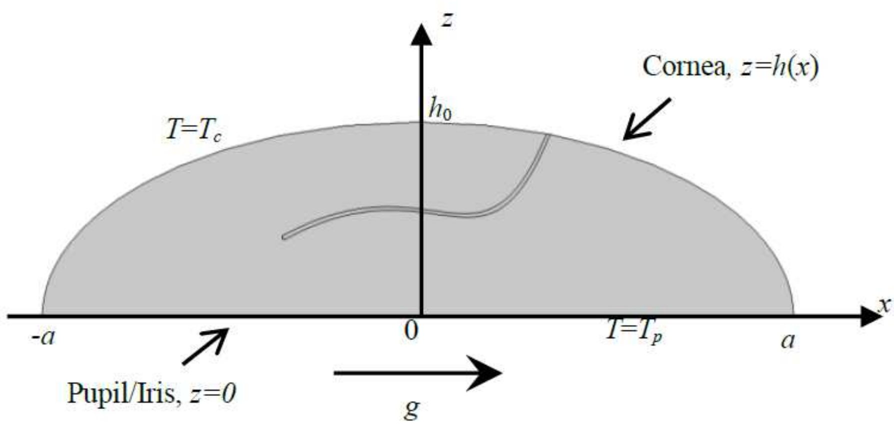

Figure 1: Schematic diagram of the DMD in the AC.

As shows in Figure 1, a two-dimensional AH flow driven by buoyancy effects in the AC during DMD in the plane $y = 0$ has been considered. The different temperature at the back of the AC, which is estimated to core body temperature (37 ^{0}C), to the outside of the cornea (say 24 ^{0}C) have induced the temperature gradient. The temperature gradient that occur across the AC of the eye imply the buoyant convection within the AC. The plane formed by pupil aperture and the iris, $z = 0$ and the anterior surface of the cornea, $z = h(x)$, which the AH flow, is introduced in a Cartesian coordinate (x, z). At the iris, the temperature is fixed at T_ρ which is 37^{0}C (the human body temperature), and the temperature at the cornea is assumed to be T_c, around 35^{0}C as a result of the cornea is cooled by the surrounding air which is estimated to be 24^0C. The gravity, g is acted along the negative x-axis as shown in Figure 1 because the patient is assumed to be in an upright position. To be realistic, a set of typical values for human eye is used: $h_0 = 2.75mm$, $a = 5.5mm$, the AH has a typical velocity of $U = 10^{-4}ms^{-1}$, $\upsilon = 0.9 \times 10^{-6}m^2s^{-1}$, gravity, $g = 9.8ms^{-2}$ and $\alpha = 3\times10^{-4}K^{-1}$ Canning et al. (2002). The AH is assumed to be Newtonian, viscous and incompressible. A DMD is assumed to be a thin and small flap attached onto the anterior surface of the cornea. According to the Boussinesq approximation, the governing equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$-\frac{\partial p}{\partial x} + \upsilon\left(\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 u}{\partial x^2}\right) + g\left(1 - \alpha\left(T - T_c\right)\right) = \rho\left(u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z}\right), \tag{2}$$

$$-\frac{\partial p}{\partial z} + \upsilon\left(\frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 w}{\partial x^2}\right) = \rho\left(u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z}\right), \tag{3}$$

$$\frac{k}{\rho C_p}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right) = u\frac{\partial T}{\partial x} + w\frac{\partial T}{\partial z}, \tag{4}$$

where υ is the kinematic viscosity, ρ is the density, k is the specific heat, C_rho is the thermal conductivity, g is the gravity and α is the coefficient of linear thermal expansion of the fluid. We assumed the thermal properties remain constant since the temperature changes involved are small, here, a standard thermal properties of water at blood temperature is used. Therefore, we have $\rho = 10^3kgm^{-3}$, $C_p = 4200\ J\ kg^{-1}K^{-1}$ and $k = 0.57\ Wm^{-1}K^{-1}$ Canning et al. (2002). The boundary conditions for the velocity are:

$$\begin{aligned} u\left(x, 0\right) = w\left(x, 0\right) = 0, \\ u\left(x, h\left(x\right)\right) = w\left(x, h\left(x\right)\right) = 0 \end{aligned} \tag{5}$$

The boundary conditions for temperature are as follows:

$$\begin{aligned} T = T_p, \quad z = 0, \\ \frac{\partial T}{\partial z} = \frac{T_c - T_p}{h_0}, \quad z = h\left(x\right). \end{aligned} \tag{6}$$

Where h_0 is a typical depth of the AC, T_c and T_p denote the temperature at the cornea and the plane formed by the pupil and the iris respectively. By assuming that the fluxes and the pressures at each point x are continuous, the pressure is known and is equal to the constant pressure $p = p_a$ at $x = a$. The deformation of the detach membrane is then computed after the accurate approximation of the flow field has been determined by applying the finite element method to (1) - (4) and subjected to (5) and (6). In order to analyze the deformation of the DMD, we assume the DMD like a thin flexible elastic plate or beam. Krichhoff theory (see, for detail, Ismail et al. (2013) and Ventsel and Krauthammer (2001)) for small deflection of thin plate is applied to estimate the deformation of the DMD under the influence of the flow. The detachment, $D(x)$ satisfies the following equation

$$D_m\left(\frac{d^4D}{dx^4}\right)=\nabla\cdot\sigma_f, \tag{7}$$

where D_m is the flexural rigidity of the DMD and is defined as

$$D_m=\frac{E_m d_m^3}{12\left(1-\upsilon_D^2\right)}.$$

E_m is the Young's modulus, d_m, is the thickness of the DMD and υ_D is the Poisson ration of the DM material. σ_f is the fluid stress tensor which obtained from the previous calculation. As usual, here we set the $E_m = 50000\text{Pa}^d$, $d_m = 10\mu m^c$ and $\upsilon_D = 0.45$ Ismail et al. (2013).

2.2 Computational Mesh and Numerical Method

The governing equations ((1) - (4) subjected to the boundary conditions (5) and (6) were solved numerically using the finite element method. Then, the computed results are used to solve equation (7). The commercial software package, COMSOL Multiphysics 5.0 was used to compute the numerical results. All the computation in the study were performed on a personal computer with a processor speed of 2.30 GHz and a RAM of 8GB. The two dimensional model was meshed using triangular elements and Lagrange quadratic polynomial was used to approximate the temperature along each surface. The displacement of the DMD was approximated by Lagrange cubic polynomial. A total 394 elements were generated by the COMSOL with a system contained total 34345 degreed of freedom.

3. Results and Discussion

The numerical results obtained in this paper have been validated by comparing the present results with the solutions obtained by Ismail and associates Ismail et al. (2013), for certain case. The great agreement between the present results with the previous obtained solutions have enhanced our confidence to the results determined in this research. The streamline and contour plot for the fluid flow in the AC driven by the buoyancy convection without the DMD is

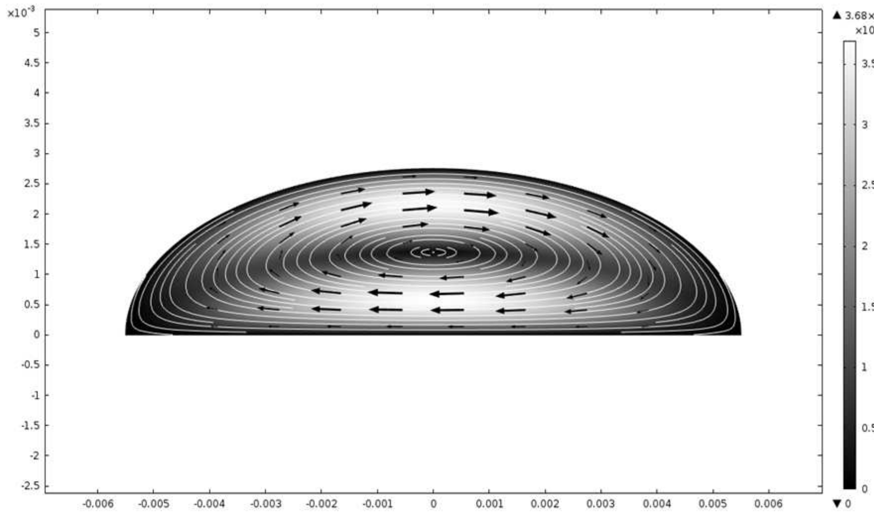

Figure 2: Streamline and contour plot for velocity profile of case without DMD.

plotted and shown in Figure 2 and is concur to the streamline plotted by Ismail and associates Ismail et al. (2013). Besides that, we have observed that the maximum flow speed computed in this study is $3.680 \times 10^{-4} ms^{-1}$ which exist at position $\left(0,\ 5.441 \times 10^{-4}\right)$ and, Ismail and associates Ismail et al. (2013), determined analytically that the maximum flow speed exist at $\left(0,\ 5.811 \times 10^{-4}\right)$ with the value $3.962 \times 10^{-4} ms^{-1}$. This illustrate that the numerical result computed in this study have a good agreement with the analytical result obtained by Ismail and associates Ismail et al. (2013). Streamline and contour plots for velocity profile of the DMD are shown in Figure 3 - Figure 6. Four vortex form in the AC by the flow that driven by the buoyant force can be observed. There is no flow can cross a streamlines so the fluid occur in between any two streamlines remain confined. Hence, the mass flow rate that cross any cross sectional slice between the two streamlines is the same at any instant in time. Base on the principle of conservation of mass, the volume flow rate per unit width between any two streamline is constant. Therefore, in order to remain the flow rate, the velocity will be increased as the decreasing of the cross sectional area between the streamline. The distance between the streamline for the region above the DMD is wider, as observed in Figure 3 - Figure 6, so the value of velocity in this area is lesser. This is consistent with the contour plot shown in the figure. The darker regions which have the lesser velocity value exist in the space between the membrane and the stroma. The movement of the fluid in the space between the membrane and the stroma may contribute

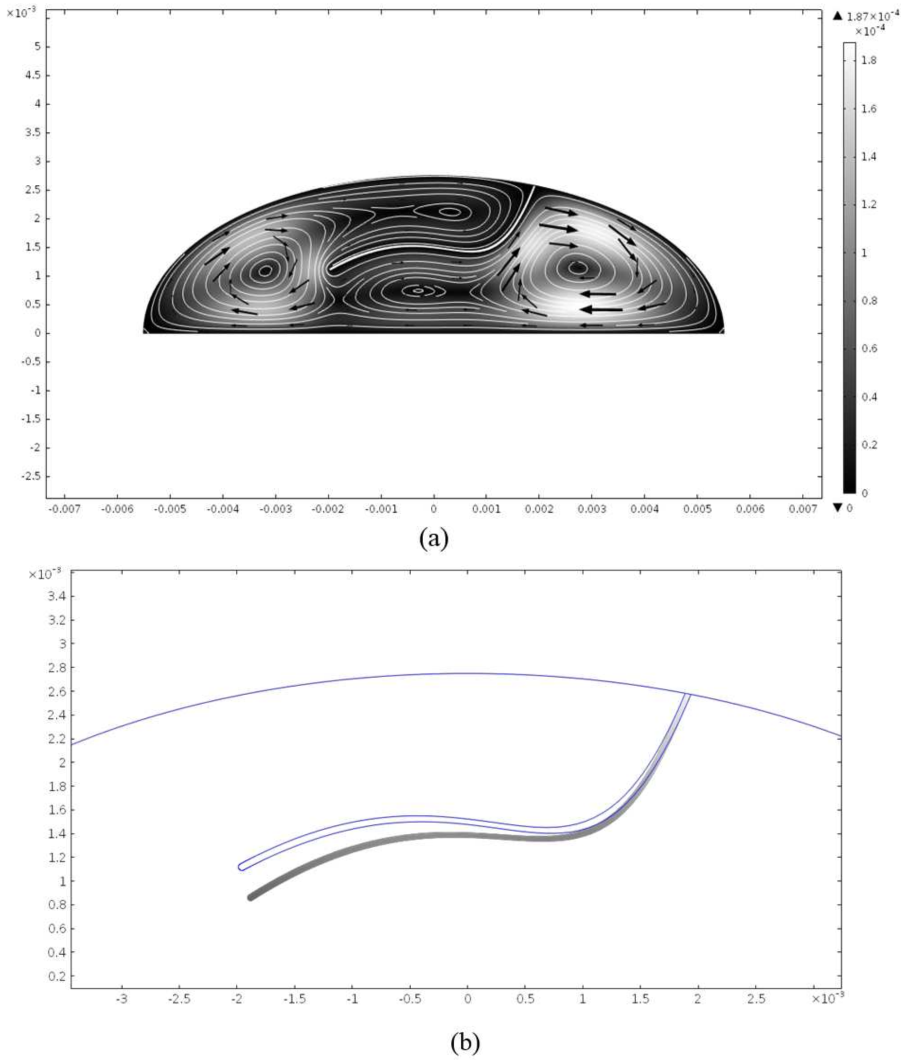

Figure 3: (a) Streamline and contour plot for velocity and (b) displacement of DMD on different position.

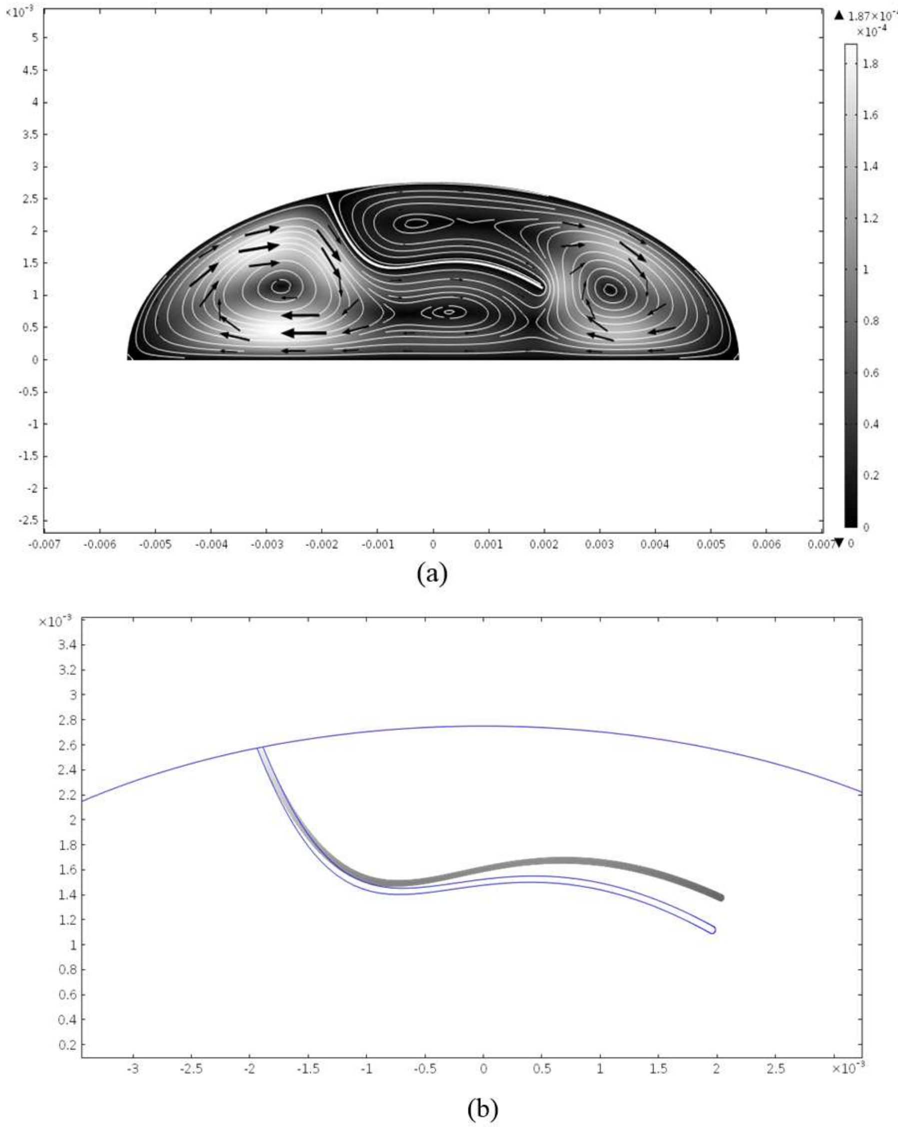

Figure 4: (a) Streamline and contour plot for velocity and (b) displacement of of DMD.

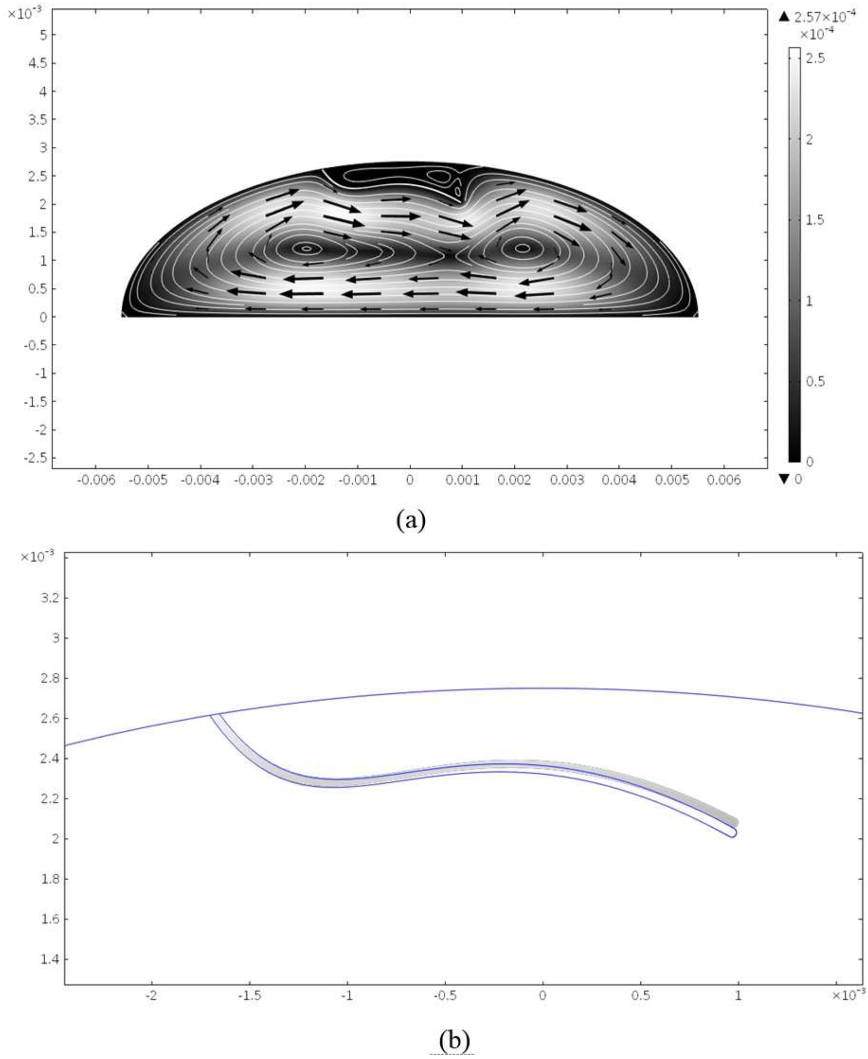

Figure 5: (a) Streamline and contour plot for velocity and (b) displacement of of trivial DMD

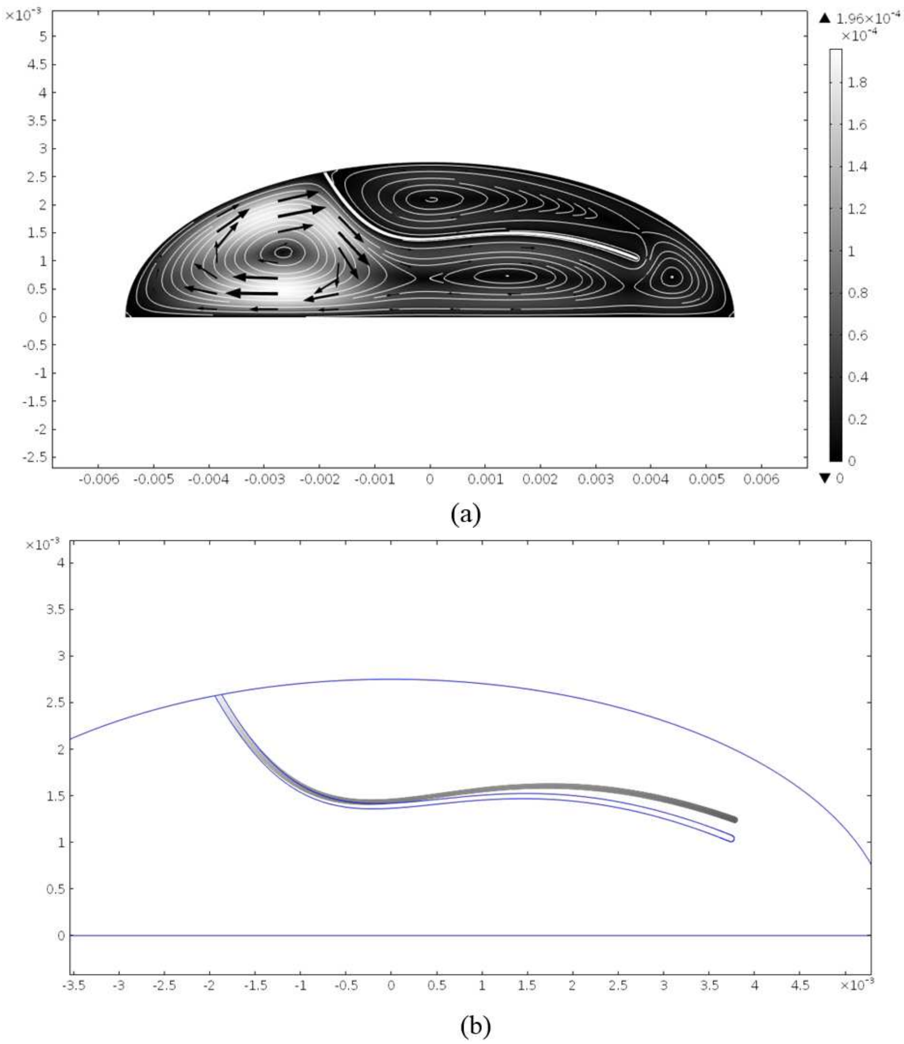

Figure 6: (a) Streamline and contour plot for velocity and (b) displacement of of elongate DMD.

to the spontaneous reattachment of the DMD. We observed from Figure 4(b), the detached membrane is pushed away from its original position and toward to the stroma. Same phenomena happen for case of trivial DMD as shown in Figure 5 and case of elongate DMD (see Figure 6). Worth mentioned, for the trivial case (see Figure 5), the displacement of the DMD is small (see Figure 5(b)) but a larger displacement is shown for the elongate case (see Fig. Figure 6(b)). This may denote that the movement of the fluid surrounding the DMD is the main key to explain the spontaneous reattachment phenomena since the gravity force is acting on the same direction. However, when the DMD exist in a different position as shown in Figure 3, the detachment of the membrane has moving away from the stroma. This may because of the patient is in an upright position and the DMD is pulled down by the gravity force which acted parallel to the x-axis (see Figure 1) and also the fluid flow on the detached membrane surface. Therefore, we can conclude that the spontaneous reattachment of DMD may happen because of the fluid flow that driven by the temperature gradient in the AC and we may state that the spontaneous reattachment of the DMD phenomena may only happen under certain conditions.

4. Summary

Numerically, the behaviour of the AH flow driven by the buoyancy force through the DMD have been studied in order to answer the question of how and why the spontaneous reattachment happen. The velocity streamline and contour for different cases are obtained and plotted. Some interesting finding of the study can be concluded as follows:

- The spontaneous reattachment of DMD may happen because of the fluid flow that driven by the temperature gradient on the surface of the DMD.
- The position of the DMD may be the main effect of the existing of the spontaneous reattachment phenomena. When the DMD exists in a different position as shown in Figure 3 the DMD move far away from the stroma.
- The length (large and small) of the DMD (see Figure 5 and Figure 6) may be also contributed to the effect of spontaneous reattachment phenomena or maybe not.

In our opinion, more research have to be done in order to fully understand the behaviour of the AC flow under the types of the DMD. The effect of the blinking of the eye may affect the temperature gradient in the AC, therefore will change the fluid flow in the AC. For further research, the effect of the blinking of the human eye have to be considered. Besides that, the mechanisms of the spontaneous reattachment of the DMD is not completely developed. How will the DMD react to the fluid flow that access on its surface and also how the fluid behaviour change based on the deformation of the DMD? This will be the next problem we are going to investigate for the aim to full realize the mechanism of the DMD under the effect of fluid flow of AC that driven by temperature gradient.

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